



Progressive Education Society's

**Modern college of Arts, Science and Commerce,
Ganeshkhind,Pune-16**

Autonomous

(Under Faculty of Science and Technology)

M.Sc.(Mathematics) : Part-II

Choice Based Credit System Syllabus

To be implemented from Academic Year 2023-2024

M.Sc. (Mathematics) : Part-II

Title of the Course: M.Sc./M.A. (Mathematics)

Preamble :

Savitribai Phule Pune University has decided to change the syllabi of various faculties from June, 2019. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects, Board of studies in Mathematics after a thorough discussion with the teachers of Mathematics from different colleges affiliated to University of Pune has prepared the syllabus of M.Sc./M. A. Semester - III and Semester- IV (w.e.f. 2020-21) Mathematics course under the Choice Based Credit System (CBCS).

The model curriculum as developed by U. G. C. is used as a guideline for the present syllabus.

Aims and Objectives of the new curriculum :

- i) To maintain updated curriculum.
- ii) To take care of fast development in the knowledge of mathematics.
- iii) To enhance the quality and standards of Mathematics Education.
- iv) To provide a broad common frame work, for exchange, mobility and free dialogue across the Indian Mathematical and associated community.
- v) To create and aptitude for Mathematics in those students who show a promise for higher studies and creative work in Mathematics.
- vi) To create confidence in others, for equipping themselves with that part of Mathematics which is needed for various branches of Sciences or Humanities in which they have aptitude for higher studies and original work.

Syllabus for M.Sc. /M.A.. Mathematics Semester III & IV

Sr. No	Courses		Credit
	Semester-III	Semester-IV	
Compulsory courses:			
1	MTUT 131: Functional Analysis	MTUT 141: Fourier Series and Boundary Value Problems	4
2	MTUT 132: Field Theory	MTUT 142: Differential Geometry	4
3	MTUT 133: Programming with Python	MTUT 143: : Introduction to Data Science	2 + 2 = 4 Theory + Practical
O=Optional Courses (Any Two)			
4	MTUTO 134: Discrete Mathematics	MTUTO 144: Number Theory	4
5	MTUTO 135: Mechanics	MTUTO 145: Algebraic Topology	4
6	MTUTO 136: Advanced Complex Analysis	MTUTO 146: Representation Theory of Finite Groups	4
7	MTUTO 137: Integral Equations	MTUTO 147: Coding Theory	4
8	MTUTO 138: Differential Manifolds	MTUTO 148: Probability and Statistics	4
9	MTUTO 139(A) : Topics in Algebra	MTUTO 149(A) : Topics in Algebra	4
10	MTUTO 139(B) : Topics in Analysis	MTUTO 149(B) : Topics in Analysis	4
11	MTUTO 139(C) : Topics in Discrete Mathematics	MTUTO 149(C) : Topics in Discrete Mathematics	4
12	MTUTO 139(D) : Topics in Applied Mathematics	MTUTO 149(D) : Topics in Applied Mathematics	4
13	MTUTO 139(E) : Topics in Geometry	MTUTO 149(E) : Topics in Geometry	4
14	MTUTO 139(F) : Skill Development Course + Project	MTUTO 149(F) : Skill Development Course + Project	2 + 2 = 4

Note : For Optional papers syllabus will be designed / framed by the respective teacher and will get it approved in the BOS meeting when that paper will be conducted in that semester

The educational tour is compulsory.

The theory and practical examination will be as per University rules and regulations.

Syllabus

Semester-III

MTUT 131: FUNCTIONAL ANALYSIS

UNIT-I: Banach Spaces

[25 Hours]

1.1 The definition and some examples.

- 1.2 Continuous linear transformations.
- 1.3 The Hahn-Banach theorem.
- 1.4 The natural imbedding of N in N^{**} .
- 1.5 The open mapping theorem.
- 1.6 The conjugate of an operator.

UNIT-II: Hilbert Spaces

[25 Hours]

- 2.1 The definition and some simple properties.
- 2.2 Orthogonal complements.
- 2.3 Orthonormal sets.
- 2.4 The conjugate space H^* .
- 2.5 The adjoint of an operator.
- 2.6 Self-adjoint operators.
- 2.7 Normal and unitary operators.
- 2.8 Projections.

UNIT-III: Finite-Dimensional Spectral Theory

[10 Hours]

- 3.1 Matrices.
- 3.2 Determinants and the spectrum of an operator.
- 3.3 The spectral theorem.
- 3.4 A survey of the situation.

Recommended Book:

G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill.
Chapters: 9, 10, 11.

Reference Books:

- 1) B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
- 2) George Bachman, Lawrence Narici, Functional Analysis, Dover Publications.
- 3) E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley, 1989.

MTUT 132: FIELD THEORY

UNIT 1: Algebraic Extension of fields:

[12 Hours]

- 1.1 Irreducible polynomials and Eisenstein criterion, adjunction of roots,
- 1.2 Algebraic extensions, algebraically closed fields.

UNIT 2: Normal and Separable extensions: [12 Hours]

- 2.1 Splitting fields,
- 2.2 Normal extensions, multiple roots, finite fields, separable extensions.

UNIT 3: Galois Theory: [20 Hours]

- 3.1 Automorphism groups and fixed fields,
- 3.2 Fundamental theorem of Galois theory,
- 3.3 Fundamental theorem of algebra.

UNIT 4: Applications of Galois theory to classical problems: [16 Hours]

- 4.1 Roots of unity and cyclotomic polynomials, cyclic extensions,
- 4.2 polynomials solvable by radicals, symmetric functions,
- 4.3 Ruler and compass constructions.

Recommended Book:

P. B. Bhattacharyya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra , Cambridge University Press, Second Edition. Chapter no: 15, 16, 17 and 18.

Reference Books:

- 1. D. Dummit and R.M.Foote, Abstract Algebra, 2nd Edition, Wiley Eastern Ltd.
- 2. T. A. Hungerford, Algebra, Graduate Texts in Mathematics, Vol. 73, SpringerVerlag, 1980 (Indian Reprint 2004).
- 3. O. Zariski and P. Sammuel, Commutative Algebra, Vol. 1, Van Nostrand. 4. I. S. Luthar, I. B. S. Passi, Algebra, Vol.
- 4, Field Theory, Narosa Publishing House
- 5. M. Artin, Algebra, Prentice Hall India, Second Edition.

MTUT 133: PROGRAMMING WITH PYTHON

UNIT-I: Introduction to Python, Python Objects [4 Hours]

- 1.1 Features of Python: Easy; Type and Run ; Syntax; Mixing; Dynamic Typing;
Built in Object Types; Numerous Libraries and Tools.
- 1.2 Chronology and Uses: Chronology; Uses.
- 1.3 Installation of Anaconda.
- 1.4 Basic Data Types Revisited: Fractions.
- 1.5 Strings.
- 1.6 Lists and Tuples: List; Tuples; Features of Tuples.

UNIT-II: Conditional Statements**[4 Hours]**

- 2.1 if, if-else, and if-elif-else constructs.
- 2.2 The if-elif-else Ladder.
- 2.3 Logical Operators.
- 2.4 The Ternary Operator
- 2.5 The get Construct.
- 2.6 Examples.

UNIT-III: Looping**[4 Hours]**

- 3.1 While.
- 3.2 Patterns.
- 3.3 Nesting and Applications of Loops in Lists.

UNIT-IV: Functions**[6 Hours]**

- 4.1 Features of a functions: Modular Programming; Reusability of Code; Manageability.
- 4.2 Basic Terminology: Name of Functions; Arguments; Return Value.
- 4.3 Definition and Invocation: Working.
- 4.4 Type of Functions: Advantage of Arguments.
- 4.5 Implementing Search.
- 4.6 Scope.
- 4.7 Recursion: Rabbit Problem; Disadvantages of Using Recursion.

UNIT-V: Iterations, Generators, and Comprehensions**[4 Hours]**

- 5.1 The Power of “For”.
- 5.2 Iterators.
- 5.3 Defining an Iterable Object.
- 5.4 Generators.
- 5.5 Comprehensions.

UNIT-VI: File Handling**[5 Hours]**

- 6.1 The File Handling Mechanism.
- 6.2 The Open Function and File Access Modes.
- 6.3 Python Functions for File Handling: The Essential Ones; The OS Methods; Miscellaneous Functions and File Attributes.

6.4 Command Line Arguments.

6.5 Implementation and Illustrations.

UNIT- VII: Strings

[5 Hours]

7.1 The Use of “For” and “While”.

7.2 String Operators: The Concatenation Operator (+); The Replication Operator; The Membership Operator.

7.3 Functions for String Handling: len(); Capitalize(); find(); count; Endswith(); Encode; Decode; Miscellaneous Functions.

UNIT-VIII: Introduction to Object Oriented Paradigm

[5 Hours]

8.1 Creating New Types.

8.2 Attributes and Functions: Attributes; Functions.

8.3 Elements of Object- Oriented Programming: Class; Object; Encapsulation; Data Hiding; Inheritance; Polymorphism; Reusability.

UNIT-IX: Classes and Objects

[5 Hours]

9.1 Defining a Class.

9.2 Creating an Object.

9.3 Scope of Data Members.

9.4 Nesting.

9.5 Constructor.

9.6 Constructor Overloading.

9.7 Destructors.

UNIT-X: Inheritance

[6 Hours]

10.1 Introduction to Inheritance and Composition : Inheritance and Methods, Composition.

10.2 Inheritance: Importance and Types: Need of Inheritance; Types of Inheritance.

10.3 Methods: Bound Methods; Unbound Method; Methods are Callable Objects; The Importance and Usage of Super; Calling the Base Class Function Using Super.

10.4 Search in Inheritance Tree.

10.5 Class Interface and Abstract Classes.

UNIT-XI: Operator Overloading

[6 Hours]

11.1 `_init_` Revisited: Overloading `_init_` (Sort of).

11.2 Methods for Overloading Binary Operators.

11.3 Overloading the += Operator

11.4 Overloading the > and < Operators.

11.5 Overloading the _boolEan_ Operators: Precedence of _bool_over _len_.

11.6 Destructors

UNIT-XII: Exception Handling

[6 Hours]

12.1 Importance and Mechanism: An exaple of Try/Catch; Manually Raising Exception.

12.2 Built In Exceptions in Python:

12.3 The Process: Exception Handling: Try/Except; Raising Exceptions.

12.4 Crafting User Defined Exceptions.

12.5 An Example of Exception Handling.

Recommended book:

H. Bhasin: Python Basics, MERCURY LEARNING AND INFORMATION Dulles, Virginia Boston, Massachusetts New Delhi Chapter 1: 1.2, 1.4, 1.5. Chapter 2: 2.2 to 2.4. Chapter 3: 3.2 to 3.7;Chapter 4: 4.2 to 4.4 Chapter 5: 5.2, to 5.8. Chapter 6: 6.2 to 6.6. Chapter 7: 7.1, to 7.6; Chapter 8: 8.1, to 8.4. Chapter 9: 9.1, 9.2, 9.3, 9.4. Chapter 10: 10.1, to 10.8.; Chapter 11: 11.1to 11.5. Chapter 12: 12.2, to 12.8.; Chapter 13: 13.2, to 13.6.

Reference Books:

1. Beginning-Python, Second Edition by Magnus Lie Hetland
2. The Complete Reference Python by Martin C. Brown
3. Head First Python by Patrick Barry
4. Learning Python, O'Reilly by Mark Lutz
5. Python in a Nutshell, O'Reilly by Alex Martelli

MTUTO 134: DISCRETE MATHEMATICS

A] Graph Theory

UNIT-I: Topics in Graph Theory

[15 Hours]

1.1 Graphs; Graphs as Models; Matrices and Isomorphism; Decomposition and Special Graphs; Degree of a vertex; Counting and Bijections.

1.2 Paths, Cycles, Trails: Connection in Graphs; Bipartite Graphs; Eulerian Circuits; Hamiltonian Cycles.

1.3 Directed Graphs: Definition and Examples; Vertex Degrees; Eulerian Digraphs.

UNIT-II: Trees [12 Hours]

- 2.1 Trees: Properties of Trees; Distance in Trees and Graphs.
- 2.2 Enumeration of Trees: Spanning Trees in Graphs; Minimum Spanning Trees; Shortest Paths; Connectivity; Edge Connectivity.

UNIT-III: Matchings [03 Hours]

- 3.1 Maximum Matchings ; Hall's Matching Condition .

B] Combinatorics

UNIT-IV: Basic Counting Principles [10 Hours]

- 4.1 Two Basic Counting Principles.
- 4.2 Simple Arrangements and Selections.
- 4.3 Arrangements and Selections with Repetitions.
- 4.4 Distributions.; Binomial Identities.

UNIT-V: Generating Functions [10 Hours]

- 5.1 Generating Functions Models
- 5.2 Calculating Coefficients of Generating Functions.
- 5.3 Partitions. ; Exponential Generating Functions.

UNIT-VI: Recurrence Relations [10 Hours]

- 6.1 Recurrence Relations Models.
- 6.2 Solutions of Linear Recurrence Relations.
- 6.3 Counting with Venn Diagrams.
- 6.4 Inclusion-Exclusion Formula.

Recommended Books:

1. Douglas B. West: Introduction to Graph Theory; 2nd Edn; PHI Learning Pvt. Ltd. Section 1.1, 1.2, 1.3 (Counting and Bijections) , Section 7.2 (Hamiltonian Cycles) Section 1.4 (Definitions, Vertex Degrees, Eulerian Digraphs) Section 2.1 (Properties of Trees; Distance) , Section 2.2 (Enumeration of Trees; Spanning Trees) Section 2.3 (Minimum Spanning Tree, Shortest Path) Section 4.1 (Connectivity, Edge-Connectivity); Section 3.1 (Maximum Matchings; Hall's Matching Condition) 2. Alan Tucker: Applied Combinatorics 6th Edn; Wiley India. Sections 5.1 to 5.5; 6.1 to 6.4; 7.1, 7.3; 8.1 to 8.2.

Reference Books:

- 1. B. Kolman, R. Busby, S.C. Ross: Discrete Mathematical Structures, 6th Edn , Pearson Edn.
- 2. John Clark, D. A. Holton: A First Look at Graph Theory, World Scientific , 1991.

MTUTO 135: MECHANICS

Unit –I : Lagrange’s Formulation**[18 Hours]**

- 1.1 Equation of Motion and conservation Theorems, Equation of Motion of a Particle, Equation of Motion of a System of Particle,
- 1.2 Conservation Theorem of Linear Momentum of the system of particles, Angular Momentum of the system of Particle, Constraint Motion, Examples of motion under constraints, Holonomic and Non – Holonomic Constraints, Scleronomic and Rheonomic Constraints, Degrees of Freedom and Generalized Co – ordinates,
- 1.3 Transformation Relations, Virtual work, Principle of Virtual Work, D., Alembert’s Principle, Conservation of Energy, Kinetic Energy as a Homogeneous quadratic function of generalized velocities, Another way of proving conservation Theorem for Energy, Lagrange’s Equations for Non – holonomic Constraints.

Unit -II: Variational Principles**[12 Hours]**

- 2.1 Generalization of Theorem, Minimum surface of revolution, Brachistochrone Problem, A case of variable end points along vertical lines $x = a$ and $x = b$, Integrant as a function of more than two dependent variables,
- 2.2 Isoperimetric problems, variational problems with moving boundaries,
- 2.3 Functional dependent on functions of two dependent variables.

Unit-III : Hamilton’s Principle**[18 Hours]**

- 3.1 Hamilton’s Principle for Non – Conservative and Conservative Systems, Configuration Space and Phase Space, Lagrange’s Equations of Motion from Hamilton’s Principle, Hamiltonian Formulation, Hamiltonian Function, Hamilton’s Canonical Equations of Motion for partially
- 3.2 Conservative and Partially Non – Conservative System, Derivation of Hamilton’s Equations of Motion from Hamilton’s Principle, Physical Meaning of the Hamiltonian,
- 3.3 Conservative and Scleronomic system, Non-conservative and Rheonomic system, partially conservative, Partially Non – conservative system, Cyclic co – ordinates in Hamiltonian, Routh’s Procedure, Principle of Least Action.

Unit IV : Two Body Central Force Motion**[12 Hours]**

- 4.1 Reduction of Two body problem to an equivalent one Body problem, Equation of Motion and the First Integral,
- 4.2 Kepler's Laws of Planetary Motion, Kepler's First second and Third Law, Deduction of Kepler's Laws, Escape velocity,
- 4.3 Newton's law of Gravitation from Kepler's Laws of Planetary Motion, Differential Equation of the orbit of a Particle, Virial Theorem

Recommended Book:

Problems in Classical Mechanics by L. N. Katkar (Narosa Publication) Chapter 1 To 4

Reference Books:

- 1) Classical Mechanics (3rd Ed.) by Herbert Goldstein, Charles Poole, John Safko (Pearson Education)
- 2) Classical Mechanics by Gupta, Kumar and Sharma (A Pragati Edition)
- 3) Classical Mechanics by Rana Joag (Mc Graw Hill India)
- 4) Classical Mechanics by R. N. Tiwari and B. S. Thakur (PHI)

MTUTO 136: ADVANCED COMPLEX ANALYSIS

Unit-I Cauchy's Theorem and it Applications: [12 Hours]

- 1.1 Revision
- 1.2 Further Applications : Sequences of holomorphic functions, Holomorphic functions defined in terms of integrals, Schwarz reflection principle, Runge's approximation theorem.

Unit-II Conformal Mappings: [20 Hours]

- 2.1 Conformal equivalences and examples: The disc and upper half plane, further examples, The Dirichlet in a strip
- 2.2 The Schwarz lemma, automorphism of the disc and upper half plane, Automorphism of the disc, Automorphism of the upper half plane

Unit-III The Riemann mapping : [20 Hours]

- 3.1 The Riemann mapping theorem, Necessary conditions and statement of the theorem, Montel's theorem, proof of the Riemann mapping theorem
- 3.2 Conformal mappings onto polygons: Some examples, The Schwarz- Christoffel integral, boundary behaviour, The mapping formula

Unit-IV An introduction to elliptic functions: [08 Hours]

4.1 Elliptic functions: Liouville's theorem, The Weierstrass function

Recommended Book

1. Stein and Shakarchi, Complex Analysis, Princeton University Press, 2006 Chapter 2 : Sections 1 to 4 Revision, Section 5, Chapter 8: Sections 1, 2, 3, 4 Chapter 9: Section 1

Reference Books :

1. J. B. Conway , Functions of one complex variable, 2nd edition, GTM 11, Springer Verlag, 1973
2. A. R Shastri, Basic complex analysis of one variable, McMillan Publishers, India, Ltd. 2011

MTUTO 137: INTEGRAL EQUATIONS

Unit-I Introductory Concepts

[12Hours]

- 1.1 Definitions
- 1.2 Classification of Linear Integral Equations
- 1.3 Solution of an Integral Equation
- 1.4 Converting Volterra Equation to ODE
- 1.5 Converting IVP to Volterra Equation
- 1.6 Converting BVP to Fredholm Equation

Unit-II Fredholm Integral Equations

[14 Hours]

- 1.7 Introduction
- 1.8 The Decomposition Method
- 1.9 The Direct Computation Method
- 1.10 The Successive Approximation Method
- 1.11 The Method of Successive Substitutions
- 1.12 Comparison between Alternative Methods
- 1.13 Homogeneous Fredholm Equations

Unit-III Volterra Integral Equations

[14 Hours]

- 1.14 Introduction
- 1.15 The Decomposition Method
- 1.16 The Series Solution Method
- 1.17 Converting Volterra Equation to IVP
- 1.18 The Successive Approximation Method
- 1.19 The Method of Successive Substitutions

1.20 Comparison between Alternative Methods

1.21 Volterra Equation of the First Kind

Unit-IV Integro-Differential Equations

[10 Hours]

1.22 Introduction

1.23 Fredholm Integro-Differential Equations

1.24 Volterra Integro-Differential Equations

Unit-V Singular Integral Equations

[10 Hours]

1.25 Definitions

1.26 Abel's Problem

1.27 The Weakly-Singular Volterra Equations

Recommnd Book:

1. Abul-Majid Wazwaz, A First Course In Integral Equations, World Scientific Publications, 1997. Chapter-1 -5.

Reference Books:

Kanwal Ram P., Linear Integral Equations, Birkhauser publication 1997. 1. Abdul J. Jerri, Introduction to Integral Equations With Applications, Wiley-Interscience; 2 edition (September 3, 1999)

MTUTO 138 : DIFFERENTIAL MANIFOLDS

Unit-I: Differential Manifolds

[25 Hours]

1.1 The Volume of a Parallelopiped.

1.2 The Volume of a Parameterized-Manifold.

1.3 Manifolds in R^n .

1.4 The Boundary of a Manifold.

1.5 Integrating a Scalar Function over a Manifold.

Unit -II: Differential Forms

[20 Hours]

2.1 Multilinear Algebra.

2.2 Alternating Tensors.

2.3 The Wedge Product.

2.4 Tangent Vectors and Differential Forms.

2.5 The Differential Operator.

2.6 The Action of a Differential Map.

Unit- III: Stokes' Theorem

[15 Hours]

3.1 Integrating Forms over Parameterized Manifolds.

3.2 Orientable Manifolds.

3.3 Integrating Forms over Oriented Manifolds.

3.4 The Generalized Stokes' Theorem.

Recommended Book:

James R. Munkres, Analysis on Manifolds (Westview Press), ISBN 0-201-31596-3. Chapter 5. Chapter 6 (except section 31). Chapter 7 (except section 36 and 38).

Reference Books:

1) Michael Spivak, Calculus on Manifolds, CRC Press (Taylor and Francis Group), Chapman and Hall Book, 1965.

2) Guillemin and Pollack, Differential Topology, AMS-Chelsea Publishing, 2010.

Semester -IV

MTUT 141 : FOURIER SERIES AND BOUNDARY VALUE PROBLEMS

Unit-I Fourier Series

[10 Hours]

1.1 Piecewise Continuous Functions, Fourier Cosine Series, Examples,

1.2 Fourier Sine Series, Examples,

1.3 Fourier Series, Examples,

1.4 Adaptations to other Intervals

Unit-II Convergence Of Fourier Series

[10 Hours]

2.1 One-Sided Derivatives, Property of Fourier Coefficients

2.2 Two Lemmas, Fourier Theorem, Discussion of the theorem and its Corollary,

Convergence on other intervals, Lemma

2.3 Absolute and uniform convergence of Fourier series, Differentiation of Fourier series,

Integration of Fourier series

Unit-III The Fourier Method

[8 Hours]

3.1 Linear Operators, Principle of Superposition

3.2 A Temperature Problem, A Vibrating String Problem

Unit-IV Boundary Value Problems

[12 Hours]

4.1 A Slab with Faces at Prescribed Temperatures, Related Problems, A Slab with Internally Generated Heat, Steady Temperatures in a Rectangular Plate

4.2 Cylindrical Coordinates, String with Prescribed Initial Conditions, Resonance, Elastic Bar

4.3 Double Fourier Series, Periodic Boundary Conditions

Unit -V Orthonormal Sets **[10 Hours]**

5.1 Inner Products and Orthonormal Sets, Examples

5.2 Generalized Fourier series, Examples

5.3 Best approximation in the Mean, Bessel's Inequality and Parseval's Equation

5.4 Application to Fourier series

Unit-VI Sturm-Liouville Problems and Applications **[10 Hours]**

6.1 Regular Sturm-Liouville Problems, Modifications, Orthogonality of Eigen functions, Real-Valued Eigen functions and Non negative Eigen Values, Methods of solution,

6.2 Examples of Eigen functions Expansions, A Temperature Problem in Rectangular Coordinates , Another Problem, Other Coordinates

6.3 Modification of the Method, Another Modification

Recommended Book:

J.W.Brown & R.V.Churchill: Fourier Series and Boundary Value Problems. VIIth Edition, McGraw Hill Education(India) Private Limited, New Delhi. Chapter-1 (Art. 1 to 8) ,Chapter-2 (Art. 9 to 18), Chapter-4 (Art. 29 to 32), Chapter-5 (Art. 34 to 43), Chapter-7 (Art. 52 to 58), Chapter-8 (Art. 59 to 69)

Reference Book:

Murray Spiegel, Fourier Analysis with Applications to Boundary Value Problems, Schaum's Outline Series, McGraw Hill.

MTUT 142: DIFFERENTIAL GEOMETRY

Unit-I **[15 Hours]**

1.1 Graphs and Level Sets,

1.2 Vector Fields,

1.3 The Tangent Space,

1.4 Surfaces.

Unit-II **[15 Hours]**

2.1 Vector Fields on Surfaces;

- 2.2 Orientation,
- 2.3 The Gauss Map,
- 2.4 Geodesics,
- 2.5 Parallel Transport.

Unit-III

[15 Hours]

- 3.1 The Weingarten Map,
- 3.2 Curvature of Plane Curves,
- 3.3 Arc Length and Line Integrals.

Unit-IV

[15 Hours]

- 4.1 Curvature of Surfaces.

Recommended Book:

J.A. Thorpe, Elementary Topics in Differential Geometry, First Indian Reprint, Springer Publication. Chapters: 1 to 12.

Reference Books:

- 1) Erwin Krysizig, Differential Geometry, Dover Publications Inc.
- 2) Christian Bar, Elementary Differential Geometry, Cambridge University Press.
- 3) Andrew Pressley, Elementary Differential Geometry, Springer.
- 4) T.J. Willmore, An Introduction to Differential Geometry, Dover Publications Inc.

MTUT 143: INTRODUCTION TO DATA SCIENCE :

1. Data science in a big data world

[08 Hours]

- 1.1. Benefits and uses of data science and big data
- 1.2. Facets of data
- 1.3. The data science process
- 1.4. The big data ecosystem and data science

2. The data science process

[10 Hours]

- 2.1. Overview of the data science process
- 2.2. Retrieving data
- 2.3. Cleansing, integrating, and transforming data
- 2.4. Exploratory data analysis

3. Machine learning **[08 Hours]**

- 3.1. What is machine learning
- 3.2. The modelling process
- 3.3. Types of machine learning
- 3.4. Semi-supervised learning

4. Handling large data **[14 Hours]**

- 4.1. General techniques for handling large volumes of data
- 4.2. General programming tips for dealing with large data sets
- 4.3. Case study Predicting malicious URLs

5. First steps in big data **[10 Hours]**

- 5.1. Distributing data storage and processing with frameworks
- 5.2. Case study: Assessing risk

6. Text mining and text analytics and Data visualization **[10 Hours]**

- 6.1. Text mining techniques
- 6.2. Data visualization options

Recommended Book:

1. Introducing Data Science, Davy Cielen, Arno D. B. Meysman, Mohamed Ali, Manning Publications Co., 1st edition, 2016

Reference Books:

- 1. An Introduction to Statistical Learning: with Applications in R, Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani, Springer, 1st edition, 2013
- 2. Introduction to Machine Learning, Ethem Alpaydin, Third Edition 2018 PHI Learning Private Limited

MTUTO 144: NUMBER THEORY

Unit I: Unique Factorization **[10 hours]**

- 1.1 : Unique Factorization in \mathbb{Z} , Unique Factorization in $k[x]$.
- 1.2 : Unique Factorization in a Principal Ideal Domain.
- 1.3 : The Rings $\mathbb{Z}[i]$ and $\mathbb{Z}[\omega]$.

Unit II: Congruence **[10 hours]**

- 2.1: Congruence in \mathbb{Z} ., The congruence $ax \equiv b(m)$.

2.2: The Chinese Remainder Theorem.

Unit III: Quadratic Reciprocity [10 hours]

3.1: Quadratic Residues.

3.2: Quadratic Reciprocity

Unit IV: Some Functions of Number Theory [10 hours]

4.1: The Greatest Integer Function.

4.2: Arithmetic Functions.

4.3: The Mobius Inversion Formula.

Unit V: Algebraic Numbers [08 hours]

5.1: Algebraic Numbers.

5.2: Algebraic Number Fields. Algebraic Integers.

5.3: Quadratic Fields.,

Recommended Books:

1. Kenneth Ireland, Michael Rosen: A Classical Introduction to Modern Number Theory, Springer, 4th Indian Reprint, 2013. Unit I: Chapter 1: Arts 1 to 4., Unit II: Chapter 3: Arts 1 to 4.

2. Ivan Niven; Herbert Zuckerman; Hugh Montgomery: An Introduction to Theory of Numbers, John Wiley and Sons, 5th Edition. Unit III: Chapter 3: Arts 3.1 and 3.2, Unit IV: Chapter 4: Arts 4.1 to 4.3, Unit V: Chapter 9: Arts 9.1 to 9.6

Reference Books:

(1) S.G.Telang: Number Theory, Tata McGraw Hill.

(2) M.B.Nathanson: Methods in Number Theory, GTM , Springer 3rd Indian Reprint, 2009.

MTUTO 145 : ALGEBRAIC TOPOLOGY

Unit-I: The Fundamental Group [25 Hours]

1.1 Homotopy of Paths.

1.2 The Fundamental Group.

1.3 Covering Spaces.

1.4 The Fundamental Group of the Circle.

1.5 Retractions and Fixed Points.

1.6 The Fundamental Theorem of Algebra.

1.7 The Borsuk-Ulam Theorem.

1.8 Deformation Retracts and Homotopy Type.

1.9 The Fundamental Group of S_n

Unit-II: The Seifert-van Kampen Theorem

[20 Hours]

2.1 Direct Sums of Abelian Groups (only revision).

2.2 Free Products of Groups (only revision).

2.3 Free Groups

2.4 The Seifert-van Kampen Theorem.

2.5 The Fundamental Group of a Wedge of Circles.

2.6 The Fundamental Groups of the Torus and the Dunce Cap.

Unit-III: Classification of Covering Spaces

[15 Hours]

3.1 Equivalence of Covering Spaces.

3.2 The Universal Covering Spaces.

3.3 Covering Transformations.

3.4 Existence of Covering Spaces.

Recommended Book:

James R. Munkres, Topology, Second Edition, Pearson Prentice Hall. Chapter 9: Sections: 51, 52, 53, 54, 55, 56, 57, 58, 59. Chapter 11: Sections: 67, 68 (Only revision), 69, 70, 71, 73. Chapter 13: Sections: 79, 80, 81, 82.

Reference Books:

- 1) Allen Hatcher, Algebraic Topology, Cambridge University Press, 2002.
- 2) M.A. Armstrong, Basic Topology, Springer International Edition, 2004.
- 3) J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
- 4) E. H. Spanier, Algebraic Topology, Springer, 1994.

MTUTO 146: REPRESENTATION THEORY OF FINITE GROUPS

Unit- I Basics of Group Theory and Linear Algebra

[8 Hours]

1.1 Group actions, General linear group, basic definitions and examples of group actions and orbits under group actions,

1.2 General linear group

Unit-II Group Representations:

[12 Hours]

2.1 Irreducible and Indecomposable representations,

2.2 Maschke's theorem and complete, reducibility, Schur's lemma

Unit-III Character Theory and Orthogonality relations: [25 Hours]

- 3.1 Orthogonality relations, characters and class functions,
- 3.2 The regular representation, permutation representation, representations of Abelian groups.

Unit-IV Fourier Analysis on Finite Groups : [15 Hours]

- 4.1 Periodic functions and Fourier transform,
- 4.2 Convolutions, Fourier Inversion,
- 4.3 Dual group.

Recommended Book :

Benjamin Steinberg, Representation Theory of Finite Groups Unit-I 2.1 to 2.3; Unit-II 3.1, 3.2; Unit-III 4.1 to 4.5; Unit-IV 5.1 to 5.3.

Reference Books:

- 1. J. P. Serre, Linear Representations of Groups
- 2. James Leibeck, Representation Theory Michael Artin, Algebra.

MTUTO 147: CODING THEORY

Unit-I Error detection: [12 Hours]

- 1.1 Correction and decoding: Communication channels, Maximum likelihood decoding,
- 1.2 Hamming distance, Nearest neighbour / minimum distance decoding, Distance of a code.

Unit-II Linear codes: [18 Hours]

- 2.1 Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes,
- 2.2 Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes,
- 2.3 Cosets, Nearest neighbour decoding for linear codes, Syndrome decoding.

Unit-III : Bounds in Coding Theory : [16 Hours]

- 3.1 Main coding theory problem, lower bounds, sphere covering bounds, Gilbert Varshamov bound, Hamming Codes,
- 3.2 Hamming bound and Perfect codes Singleton bound and MDS codes

Unit-IV Cyclic codes: [10 Hours]

- 4.1 Definitions, Generator polynomials,
- 4.2 Generator and parity check matrices,
- 4.3 Decoding of cyclic codes, Burst-error-correcting codes.

Unit-IV Some special cyclic codes:

[04 Hours]

- 5.1 BCH codes, Definitions, Parameters of BCH codes.

Recommended Book:

San Ling and Chaoxing, Coding Theory- A First Course (Cambridge University Press, 2004) (Sections 2.1, to 2.5); (Sections 3.1. to 3.4 and Sections 4.1, to 4.8); (Sections 5.1, to 5.4) , (7.1, to 7.4); (8.1, 8.1.1 and 8.1.2)

Reference Books:

- 1) San Ling and Chaoxing, Coding Theory- A First Course
- 2) Raymond Hill, A First Course in Coding Theory (Oxford)
- 3) Lid and Pilz, Applied Abstract Algebra Second Edition

MTUTO 148: PROBABILITY AND STATISTICS

Unit I: Introduction to Probability

[5 lectures]

- 1.1 Sample space, events, probability of an event, additive rules, conditional probability,
- 1.2 Multiplicative rule, Bayes' rule.

Unit II: Random Variable

[16 lectures]

- 2.1 Concept of a random variable, discrete probability distribution, continuous probability distribution, joint probability distribution,
- 2.2 Independent random variables, Chebyshev's theorem, Mean of a random variables, Variance and Covariance, Mean and Covariance of linear combinations of random variables, Functions of random variables, transformations of variables,
- 2.3 Moments and Moment Generating Functions, definition of Expectation, theorems on Expectation and its related problems, Variance in terms of Expectation and related problems,
- 2.4 Covariance in terms of Expectation and related problems, Variance of a Linear Combination.

Unit III: Some Discrete Probability Distributions

[12 lectures]

- 3.1 Binomial and Multinomial distributions, Hypergeometric distribution, Negative Binomial distribution, Geometric distribution,
- 3.2 Poisson distribution and Poisson process.

Unit IV: Some Continuous Probability Distributions

[13 lectures]

4.1 Continuous Uniform distribution, Normal distribution, area under the normal curve, applications of the Normal distribution, normal approximation to the binomial distribution,

4.2 Gamma distribution, Exponential distribution, Chi-squared distribution.

Unit V: Linear Regression and Correlation

[14 lectures]

5.1 Simple Linear Regression, lines of Regression, Least Squares and Fitted Model,

5.2 properties of Least Squares Estimators, Regression Coefficients, choice of Regression model, data plot and transformations,

5.3 Karl Pearson's Coefficient of Correlation, Properties and Problems, Spearman's Rank Correlation Coefficient, Method of Concurrent Deviations.

Recommended Book:

□ Probability and Statistics for Engineers and Scientists, by R. Walpole, R.H. Myers, S.L. Myers, and K. Ye (Seventh Edition, Pearson India). Chapters: 2, 3, 4, 5, 6.1 – 6.8, 7.3, 11.

Reference Books:

□ Introduction to Probability and Statistics for Engineers and Scientists, by Sheldon M. Ross (Fourth Edition).

□ A first course in Probability, by Sheldon M. Ross (Ninth Edition).

□ Mathematical Statistics, by Parimal Mukhopadhyay.

□ Statistics for the Life Sciences, by M. Samuels, J. Witmer and A. Schaffner (Fifth Edition, Pearson India)

□ Probability and Statistics for Engineers, by Richard Gupta, C B Gupta.